THEORETICAL UNCERTAINTIES IN THE DESCRIPTION OF THE NUCLEON–DEUTERON ELASTIC SCATTERING AT $E = 135$ MeV*

Yuriy Volkotrub, Roman Skibiński, Jacek Golak
Kacper Topolnicki, Henryk Witała

M. Smoluchowski Institute of Physics, Jagiellonian University
Łojasiewicza 11, 30-348, Kraków, Poland

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The elastic nucleon–deuteron (Nd) scattering process at nucleon laboratory energies up to 200 MeV can be used to study the three-nucleon interaction ($3N$). In order to draw final conclusions based on the comparison of theoretical predictions and data, an estimation of theoretical uncertainties is necessary. We focus here on the statistical uncertainties of theoretical predictions. We use the One-Pion-Exchange Gaussian (OPE-Gaussian) nucleon–nucleon (NN) potential and compare our predictions for the $3N$ observables with results based on the AV18 potential and results obtained with chiral potentials as well as with available data. We give examples of polarization observables at $E = 135$ MeV and discuss magnitudes of some theoretical uncertainties.

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1. Introduction — formulation of the problem

The determination of uncertainties in theoretical calculations for the nuclear observables is a very timely issue. It is clear that the necessity of reliable theoretical errors estimation stems from the growing precision of experimental data as well as from the fact that nowadays we strive to study details of the nuclear Hamiltonian. Investigations of the nuclear interaction in the elastic Nd scattering are a good example: the two-nucleon ($2N$) potential is relatively well-known and the details of the $3N$ force are still not well-determined.

There are different types of theoretical uncertainties of the elastic Nd scattering observables [1]. Here, we focus on the statistical errors arising from a propagation of uncertainties of parameters of the $2N$ interaction to

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the $3N$ system and on the uncertainties arising from using the various models of nuclear interaction. In this contribution, we supplement our earlier studies [1, 2] and refer the reader to these papers for a more general discussion. While in [1] $3N$ elastic scattering observables at 13, 65 and 200 MeV have been shown, in [2] an additional example of the nucleon–nucleon spin correlation coefficient $C_{z,x}$ (again at $E = 13, 65$ and 200 MeV) obtained within the Faddeev approach [3] is given.

The knowledge of the correlation matrix of the NN potential parameters allows us to perform a statistical analysis of the considered observables. Thereby, it is possible to investigate, for instance, correlations between observables and to analyze the theoretical uncertainties of predictions in few-nucleon systems which arise from the propagation of uncertainties of $2N$ potential parameters. Currently, a few models of the NN interaction are constructed in such a way that the correlation matrix of their parameters is known. We use the One-Pion-Exchange (OPE)-Gaussian NN interaction derived by the Granada group [4, 5] and the N^{4}LO chiral force with semilocal regularization in momentum space derived by the Bochum–Bonn group [6]. In the following, we use the N^{4}LO interaction with the regularization parameter $\Lambda = 450$ MeV. The structure of the OPE-Gaussian force is similar to the structure of the standard semi-phenomenological AV18 model [7]. We use the Faddeev approach to calculate $3N$ observables for the elastic Nd scattering. This framework as well as our numerical performance is described in details *e.g.* in [3]. In the present work, we neglect a $3N$ interaction and apply only the $2N$ force, which enters the Faddeev equation via the t — matrix operator. Solutions of the Faddeev equation are used to obtain the transition amplitude for elastic Nd scattering, from which any observable for this process can be calculated. Our numerical solution is obtained by using $3N$ partial waves, and we take into account all states with the two-body total angular momentum j up to $j_{\max} = 5$ and the three-body total angular momentum J up to $J_{\max} = \frac{25}{2}$. We refer the reader to [8–12] for a discussion of the role of $3NF$s in elastic Nd scattering.

2. Results for the 3N observables

We have chosen the spin correlation coefficient $C_{z,x}$ and the spin transfer coefficient $K_y^{y'}(N)$ to exemplify magnitudes of statistical uncertainties. In Fig. 1, we show predictions for the spin correlation coefficient $C_{z,x}$ and the spin transfer coefficient $K_y^{y'}(N)$. For both observables, the difference between the AV18 predictions and the OPE-Gaussian results is especially big already at the incoming nucleon laboratory energy $E = 135$ MeV. The cyan band comprising 34 predictions (what corresponds to $\Delta_{68\%}$ estimator, see [1] and [2]) shows the statistical uncertainty for the predictions based

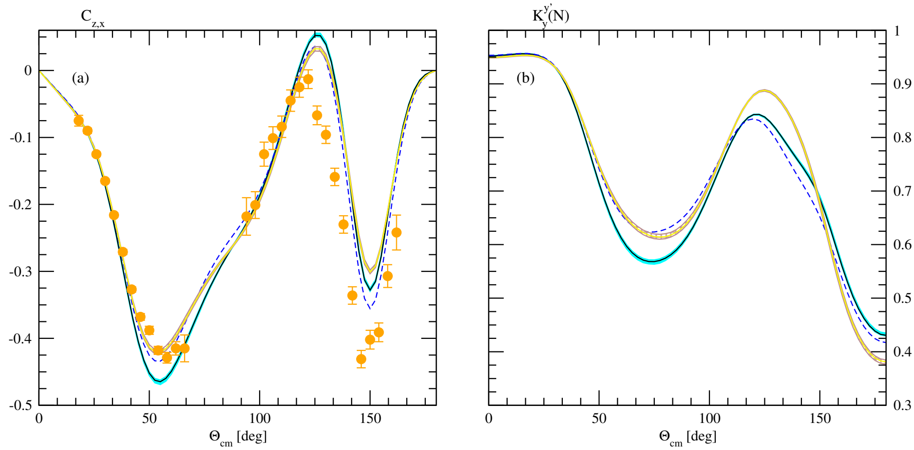

Fig. 1. (Color online) The Nd elastic scattering spin correlation coefficient $C_{z,x}$ (a) and the nucleon to nucleon spin transfer coefficient $K_y^{y'}$ (b) at the incoming nucleon laboratory energy $E = 135$ MeV as a function of the c.m. scattering angle $\theta_{\rm cm}$. The black curve represents predictions obtained with the OPE-Gaussian force, the cyan band shows its statistical uncertainty, the yellow curve represents predictions obtained with the chiral N^{4}LO ($\Lambda = 450$ MeV) potential, the lighter/brown band shows its statistical uncertainty, and the dashed/black curve represents predictions based on the AV18 force. The data are from [13].

on the OPE-Gaussian potential. The black curve represents predictions obtained with the central values of the OPE-Gaussian force parameters. The difference between the two predictions (the OPE-Gaussian and the AV18) amounts to $\approx 3.5\%$ at the minimum of $C_{z,x}$, while the statistical error of the OPE-Gaussian results is only 0.35%. For $K_y^{y'}$, these values amount to $\approx 6\%$ and $\approx 0.4\%$, respectively. In Fig. 1, we also show the lighter/brown band representing the $\Delta_{68\%}$ estimator based on 34 predictions of the chiral N^{4}LO ($\Lambda = 450$ MeV) potential with semilocal momentum space regularization and predictions based on the central values of the chiral potential parameters (the yellow curve). The statistical uncertainties of theoretical predictions remain smaller than the dominant uncertainty arises from a dispersion due to using various models of nuclear interaction.

3. Summary

Summarizing, we have successfully applied the OPE-Gaussian force and the chiral N^{4}LO potential with the semilocal momentum space regularization to study the propagation of uncertainties of $2N$ interaction parameters to $3N$ observables. We conclude that for the spin correlation coefficient $C_{z,x}$

and for the spin transfer coefficient $K_y^{y'}$, the resulting statistical uncertainty is small, both for the OPE-Gaussian potential and the chiral N^{4}LO ($\Lambda = 450$ MeV) interaction. This is similar to results for other spin observables discussed in [1] and [2]. The description of data delivered by the OPE-Gaussian force and the chiral N^{4}LO potential with the semilocal momentum space regularization is in a quantitative agreement with the picture obtained using the AV18 model. We conclude that the uncertainty arising from using various models of the nuclear interaction is greater than the statistical error. These conclusions agree with results of a more extended analysis given in [1].

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